

Study on Plasticity Modification for Measuring High Residual Stress by Hole-Drilling Method

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Abstract: Hole-drilling method is a commonly used method for measuring residual stress. The calibration coefficients in ASTM E837-13a would cause large errors due to the plasticity deformation of materials. In the study, calibration coefficients were modified in the plasticity deformation stage based on the distortion energy theory. The calibration experiment of calibration coefficients was simulated by the finite element model, and the plasticity modification formulas of 7075 aluminum alloy were obtained. From the results of uniaxial tensile loading test, the measuring errors of high residual stress are significantly reduced from -4.071% ~ 53.440% to -5.140% ~ 0.609% after the plasticity modification. This work provides an effective way to expand the application of hole-drilling method.

Key words: Calibration Coefficients; Finite Element Analysis; Plasticity Modification; Hole-Drilling Method; Residual Stress

1 Introduction

Residual stress is caused by the machining such as casting, cutting, welding and forming, which is the key factor to determine the machining accuracy, fatigue strength and dimensional stability of parts. Hole-drilling method is a commonly used method for measuring residual stress proposed by Mathar in 1934^[1-3]. Traditional hole-drilling method is mainly used in uniform or non-uniform residual stress measurement of isotropic linear elasticity materials^[2, 4]. Hole-drilling method is easy to use and less destructive to specimens. The experimental results are reliable and can be applied to the residual stress measurement of the various engineering materials^[3, 5, 6]. At the same time, the relevant standard ASTM E837-13a has been established for hole-drilling method.

In 1966, Rendler and Vigness^[7] introduced calibration coefficients A and B into hole-drilling method, which could be used to measure the residual stress of isotropic materials. However, according to the theoretical analysis of Kirsch^[8], calibration coefficients A and B are related to the properties of the materials, and different materials have different calibration coefficients A and B , which needed to be

calibrated separately. In 1981, Schajer^[9] introduced dimensionless calibration coefficients $\bar{a}$ and $\bar{b}$, which were only related to the geometric shape of the materials, but not to the properties of the materials. And $\bar{a}$ and $\bar{b}$ could be obtained by the theoretical calculation or mechanical tensile calibration test. Therefore, they are universally applicable and have the research value.

In recent years, scholars have done some study on calibration coefficients $\bar{a}$ and $\bar{b}$. Zheng et al^[10] analyzed the effects of the specific aperture, drilling depth and specimen thickness on calibration coefficients $\bar{a}$ and $\bar{b}$, and extended the usable range of them to a certain extent. Nau et al^[11] studied the influence of Poisson's ratio on calibration coefficients $\bar{a}$ and $\bar{b}$. Because the stress calculation formulas of hole-drilling method are derived from the elasticity hypothesis, if high residual stress is measured and calculated according to the traditional method, the results obtained are larger than the actual value, even exceeding the tensile strength of the materials, which is due to the plasticity deformation at high residual stress. Therefore, it is necessary to modify the

measured value of high residual stress and reduce the measuring errors between the modified value and the actual value. In the measurement of high residual stress, it is an effective method to reduce the stress measuring errors by modifying calibration coefficients.

In this study, calibration coefficients $\bar{a}$ and $\bar{b}$ in ASTM E837-13a were modified in the plasticity deformation stage, and the plasticity modification formulas were given. The finite element simulation model of 7075 aluminum alloy in the through-hole case was established to simulate the calibration experiment of calibration coefficients. The validity of the method was verified by comparing the measuring errors of high residual stress before and after modification.

2 Plasticity Modification Method of Calibration Coefficients for Measuring High Residual Stress by Hole-Drilling Method

2.1 The Principle of Measuring Residual Stress by Hole-Drilling Method

The hole-drilling method can be used to measure the isotropic residual stress of the linear elasticity materials, which is a method of measuring the strain released before and after drilling on the surface of the specimens by the strain gage rosette, and then calculating the residual stress at the edge of the drilled hole. In the study, a type A strain gage rosette was used, as shown in Fig.1.

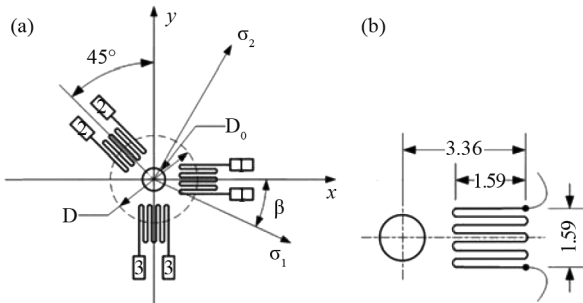


Fig. 1 (a) Schematic Geometry of a Type a Strain Gage Rosette; (b) Detail of the Strain Gage Rosette

In Fig.1, D is the central circle diameter of the

strain gage rosette, and the value of it is 5.13 mm. D_0 is the diameter of the hole, β is the angle between the maximum principal stress and the x -axis, the directions of the strain gage grid 1 and strain gage grid 3 coincide with the x -axis and y -axis respectively, and the angle between the strain gage grid 2 and the y -axis is 45° . The strains at the strain gage grid 1, 2 and 3 are respectively ε_1 , ε_2 and ε_3 . When the direction of the strain gage grid 1 coincides with the principal stress direction, β is 0° at this moment, and the residual stress inside the specimens can be obtained ^[10]:

$$\begin{aligned}\sigma_1 &= -\frac{E(\varepsilon_3 + \varepsilon_1)}{2\bar{a}(1 + \nu)} + \frac{E|\varepsilon_3 - \varepsilon_1|}{2\bar{b}} \\ \sigma_2 &= -\frac{E(\varepsilon_3 + \varepsilon_1)}{2\bar{a}(1 + \nu)} - \frac{E|\varepsilon_3 - \varepsilon_1|}{2\bar{b}}\end{aligned}\quad (1)$$

In equation (1), σ_1 is the maximum principal stress and σ_2 is the minimum principal stress, E is Young's modulus, ν is Poisson's ratio, $\bar{a}$ is the calibration coefficient for isotropic stress, $\bar{b}$ is the calibration coefficient for shear stress.

2.2 Calibration Calculation of Calibration Coefficients of Hole-Drilling Method

The calibration process of calibration coefficients is as follows: After the strain gage rosette is attached to the specimens and the load is applied on the specimens, the strain values ε''_1 , ε''_2 and ε''_3 of the specimens before drilling are measured first, and then the strain values ε'_1 , ε'_2 and ε'_3 of the specimens after drilling are measured. Finally, the strain values ε_1 , ε_2 and ε_3 due to the stress released can be derived by using equation (2):

$$\begin{aligned}\varepsilon_1 &= \varepsilon''_1 - \varepsilon'_1 \\ \varepsilon_2 &= \varepsilon''_2 - \varepsilon'_2 \\ \varepsilon_3 &= \varepsilon''_3 - \varepsilon'_3\end{aligned}\quad (2)$$

In the case of uniaxial stress, σ_1 is equal to the calibration load value applied along the direction of the strain gage grid 1 when calibrating, and $\sigma_y = 0$ at the moment. Then calibration coefficients $\bar{a}$ and $\bar{b}$ can be deduced:

$$\begin{aligned}\bar{a} &= \frac{2E}{1+v} \frac{\varepsilon_1 + \varepsilon_3}{2\sigma_x} \\ \bar{b} &= 2E \frac{\varepsilon_1 - \varepsilon_3}{2\sigma_x}\end{aligned}\quad (3)$$

2.3 Plasticity Modification Method for High Residual Stress Measurement

Because the stress calculation formulas of hole-drilling method are derived according to the elasticity hypothesis, when plasticity deformation occurs, it will cause large errors. Therefore, it is necessary to modify the stress calculation formulas of hole-drilling method. In the study, the distortion energy theory was used to modify the plasticity effect under the plane stress. The distortion energy theory assumes that the material yield is caused by the distortion energy density v_d . No matter what kinds of stress states in the materials are, as long as v_d at any point in the materials reaches the limit value v_{du} of the materials, the materials at this point will produce plasticity yield^[12]. In the case of the plane stress, the equation of v_d is shown in equation (4):

$$v_d = \frac{1+v}{3E} [\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2] \quad (4)$$

According to Hooke's law, the following relationship of the stress and strain can be obtained:

$$\begin{aligned}\varepsilon_1 &= \frac{1}{E} (\sigma_1 - \nu\sigma_2) \\ \varepsilon_3 &= \frac{1}{E} (\sigma_2 - \nu\sigma_1)\end{aligned}\quad (5)$$

The following equation can be obtained from the simultaneous equations (4) and (5):

$$v_d = \frac{SE}{3(1+\nu)(1-\nu)^2} \quad (6)$$

In equation (6):

$$S = (1 + \nu^2 - \nu)(\varepsilon_1^2 + \varepsilon_3^2) - (1 + \nu^2 - 4\nu)\varepsilon_1\varepsilon_3 \quad (7)$$

In equation (7), S is a strain-related parameter, which reflects the deformation of the materials in all directions and the distortion energy at the corresponding strain state. Therefore, the value of S is the criteria of whether the plasticity deformation occurs.

In plasticity modification, the strain corresponding to the applied load is first obtained, and then calibration coefficients and the value of S in the corresponding case are obtained. The relationship among $\bar{a}$, $\bar{b}$ and S is established by the graph, and the change of $\bar{a}$ and $\bar{b}$ with S is analyzed. Finally, combining with the obtained graph, the functional relationship among $\bar{a}$, $\bar{b}$ and S is obtained by least-squares linear fitting, which is the plasticity modification equation.

3 Plasticity Modification of Calibration Coefficients Based on the Finite Element Numerical Simulation

3.1 Finite Element Modelling

In this study, the calibration process of calibration coefficients was simulated by ANSYS. The simplified elastic-plastic stress-strain curve was referenced in the simulation, and the bilinear model was used in the simulation. As shown in Fig.2, the entire two-dimensional models of the specimen before and after drilling were simulated by ANSYS. The PLANE82 element was used in the simulation, with 47859 units and 143927 nodes. In the simulation, the uniform load was applied to the left and right sides of the models in both positive and negative directions of the x -axis. At the same time, a through-hole with a diameter of 1.52 mm (0.3 D) was designed in the models, and the corresponding calibration coefficients in ASTM E837-13a were $\bar{a} = 0.088$, $\bar{b} = 0.283$. The relevant parameters of 7075 aluminum alloy are as follows: Young's modulus $E = 71$ GPa, Poisson's ratio $\nu = 0.33$, plasticity tangent modulus $E_p = 2.5$ GPa, density $\rho = 2800$ kg/m³ and yield strength $\sigma_s = 290$ MPa. In order to make the simulation results more accurate, the distance between the center of the strain gage rosette and the edge of the specimen should be larger than 12 D when using hole-drilling method to measure the stress^[10]. Therefore, the size of the simulation model was 100 mm $\times$ 100 mm.

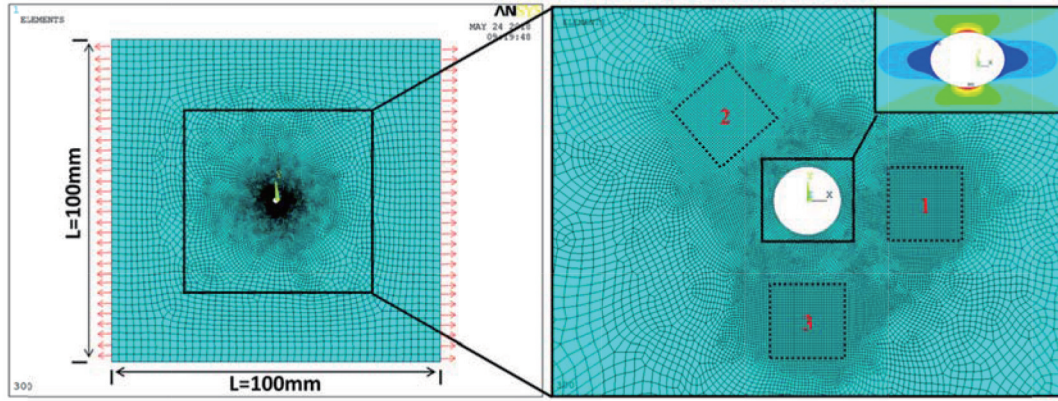


Fig. 2 Two-Dimensional Simulation Model After Drilling and Stress Distribution at the Edge of the Hole

3.2 Finite Element Numerical Simulation of Calibration Coefficients

After the simulation was completed, the strain in the x -axis direction of the corresponding nodes at the strain gage grid 1 before and after drilling was extracted, and the strain average values ε''_1 and ε'_1 were calculated. The strain in the y -axis direction of the corresponding nodes at the strain gage grid 3 before and after drilling was extracted, and the strain average values ε''_3 and ε'_3 were calculated. Substituting the strain values obtained before and after drilling into equation (3) could calculate the values of $\bar{a}$ and $\bar{b}$. Fig.3 shows the variation of calibration coefficients $\bar{a}$ and $\bar{b}$ under different load conditions. In Fig.3, it can be seen that when the load is less than $1/3 \sigma_s$ (about 100 MPa), calibration coefficients $\bar{a}$ and $\bar{b}$ change slightly in the elasticity deformation stage, and the average values of calibration coefficients $\bar{a}$ and $\bar{b}$ of simulation calibration are $\bar{a} = 0.08638$, $\bar{b} = 0.26263$. By comparing the calibrated calibration coefficients with the calibration coefficients in ASTM E837-13a ($\bar{a} = 0.088$, $\bar{b} = 0.283$), it can be seen that the error between them is small and the consistency is good, which shows the reliability of the simulation. When the load is greater than $1/3 \sigma_s$ (about 100 MPa), the plasticity effect will occur, and the changes of $\bar{a}$ and $\bar{b}$ are large, and both increase linearly with the increase of the load,

and the slopes of them can be obtained by fitting analysis. At this time, if plasticity modification of calibration coefficients is not conducted, a large error will be caused, so it is necessary for the plasticity modification.

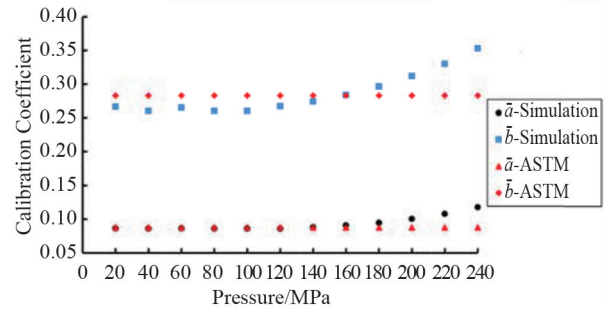


Fig. 3 The Variation Curve of Calibration Coefficients Under Different Calibration Loads

3.3 Plasticity Modification Of Calibration Coefficients In High Residual Stress Measurement

Calibration coefficients can be modified by combining the strain values obtained from the simulation with the plasticity modification method in the study. The ε_1 and ε_3 obtained by the simulation are substituted into equation (7) to obtain the corresponding the value of S , and the relationship among $\bar{a}$, $\bar{b}$ and S can be obtained by least-squares linear fitting, as shown in Fig.4.

The modified calibration coefficients equation (8) can be obtained from Fig.4, where S is the critical value from elasticity deformation to plasticity de-

formation. In equation (8), it can be seen that $\bar{a}$ and $\bar{b}$ are approximately constants at the elasticity stage, and the change of them is approximately an oblique line at the plasticity stage. In the practical application, the value of S is calculated first and then substituted into equation (8). According to the comparison between the value of S and 56832, it is determined whether to conduct the plasticity modification on calibration coefficients. Finally, the obtained calibration coefficients are substituted into the stress calculation formulas to obtain the stress.

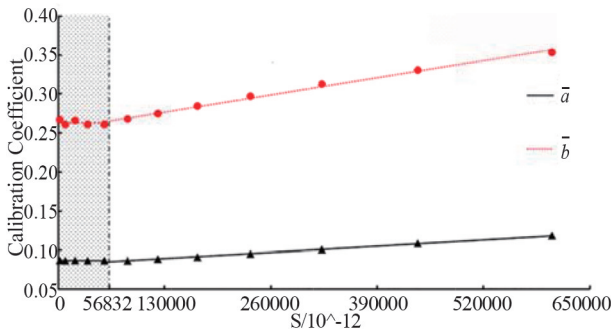


Fig. 4 The Relation Between Calibration Coefficients and the Parameter S

$$\bar{a} = \begin{cases} 0.088, S \leq 56832; \\ 6 \cdot 10^{-8}S + 0.0812, S > 56832 \end{cases} \quad (8)$$

$$\bar{b} = \begin{cases} 0.283, S \leq 56832; \\ 6 \cdot 10^{-7}S + 0.254, S > 56832 \end{cases}$$

3.4 Comparison of Stress Calculation Results Before and After Plasticity Modification of Calibration Coefficients

Combining with the simulated strain values and calibration coefficients before and after plasticity modification, the stress values are calculated and compared. Fig.5 shows the measuring errors between the calculated load and the calibrated load by using calibration coefficients before and after the plasticity modification, which are the stress calculation result by the simulation. In Fig.5, it can be seen that in the case of no plasticity modification, the measuring errors in the elasticity stage (the value of S is less than 56832) are small, and the variation range of the measuring errors is $-4.996\% \sim -3.720\%$. Besides,

the measuring errors are negative, which indicates that the calculated load is less than the calibration load. But in the plasticity stage (the value of S is greater than 56832), due to the existence of plasticity effect, the measuring errors are large and the variation range of the measuring errors is $-4.972\% \sim 29.398\%$. Besides, the error value changes from negative to positive and increases with the increase of the load. In the case of plasticity modification, the measuring errors after plasticity modification are obviously reduced and the variation range of the measuring errors is $-4.972\% \sim -0.888\%$. The study shows that the plasticity modification method is effective.

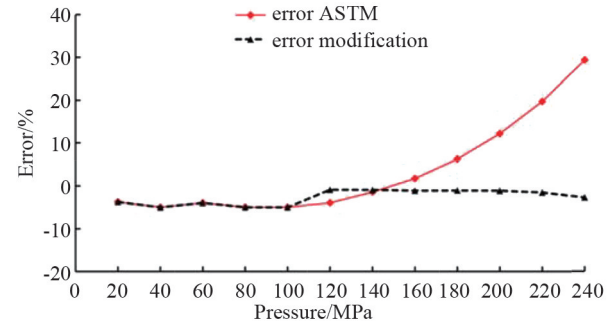


Fig. 5 Comparison of Stress Measuring Errors Before and After Plasticity Modification in the Simulation

4 The Verification Experiment of Plasticity Modification of Calibration Coefficients

4.1 Tensile Test Specimens Making

The study was verified by the tensile loading test. As shown in Fig.6, tensile test specimens with a total length of 175 mm, a thickness of 1 mm, and a width of 15 mm, are made according to the metal tensile test standard. The material of tensile test specimens is 7075 aluminum alloy, and the heat treatment state is T6. The chemical composition of 7075 aluminum alloy is shown in Table 1. At room temperature, the stress-strain curve of 7075 aluminum alloy specimens in three tensile tests conforms to the simplified elastic-plastic stress-strain curve, which indicates that the experimental aluminum alloy mate-

rial has high deformation repeatability under the same load and increases the accuracy of the experiment.

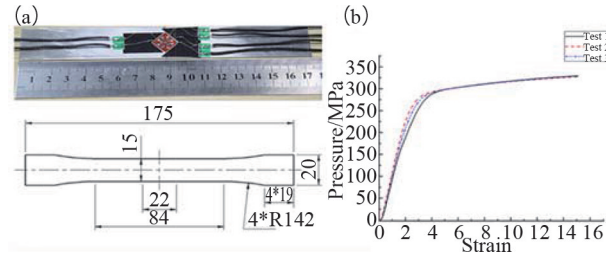


Fig. 6 (a) The Cad Drawings and Physical Model of Tensile Test Specimens; (b) The Elastic-Plastic Stress-Strain Curve of 7075 Aluminum Alloy in the Tensile Loading Test

Table 1 Chemical Composition of 7075 Aluminum Alloy Specimens

Si	Fe	Cu	Mn	Mg	Cr	Zn	Ti	Al
0.4	0.5	1.2-2.0	0.3	2.1-2.9	0.18-0.28	5.1-6.1	0.2	others

4.2 Platform of Verification Experiment for Plasticity Modification

Considering that the experiment is conducted under the uniaxial stress conditions, the uniaxial tensile loading test is used to verify the derived formulas. As shown in Fig.7, the verification experiment system of uniaxial tensile loading test was constructed by the electronic universal testing machine (WDW-10) and drilling device (HK21B). In the experiment, the room temperature was 20°C and the loading rate was 0.1 kN/s. The tensile test specimens were

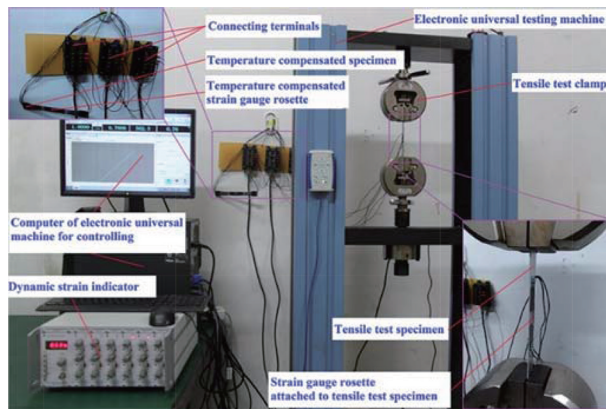


Fig. 7 The Verification Experiment System of Uniaxial Tensile Loading Test

clamped on the electronic universal testing machine. The strain gage rosette was attached to the specimens and the temperature compensated strain gage rosette was connected to the bridge with the terminal YE29003A. In order to eliminate the influence of the strain caused by the drilling, the bridge was again adjusted before measurement.

4.3 Analysis of Results

The verification experiment system of uniaxial tensile loading test was used to verify the plasticity modification. In the experiment, a uniaxial load was applied to tensile test specimens before drilling. The load was increased from 30 MPa to 250 MPa, each time increased by 10 MPa. When the strain reading was stable, the strain values ε''_1 , ε''_2 and ε''_3 were recorded. Then the same tensile loading test was conducted on the drilled tensile specimens to obtain the strain values ε'_1 , ε'_2 and ε'_3 . The strain values were measured by the dynamic strain indicator YE3817C, and the values of ε_1 , ε_2 , ε_3 and S could be obtained using the measured strain values. Then the relevant values were substituted into equation (8) to obtain the modified calibration coefficients, and the calculation load was further obtained. Finally, comparing the calculation load with the calibration load to obtain the error between them. Using this method to average multiple sets of experimental data, the experimental results obtained are shown in Fig.8. From Fig.8, it can be seen that the measuring errors of 7075 aluminum alloy are obviously reduced after the plasticity modification in the plasticity deformation stage (the value of S is greater than 56832). The variation range of the average value of the measuring errors before the plasticity modification is -4.071% ~ 53.440%, and the variation range of the average value of the measuring errors after the plasticity modification is -5.140% ~ 0.609%. Therefore, the plasticity modification is proved to be effective by experiments.

It should be pointed out that it is inevitable to

introduce a certain degree of the error in the drilling experiment, and when the calibration load is small, the error introduced in the experiment will have a greater impact on the results. Therefore, the experimental results when the load is less than 30 MPa are not included in the discussion. At the same time, the method adopted in this study is the way of loading after drilling, and compared with the way of drilling after loading, the error is small, so this effect can be ignored^[13].

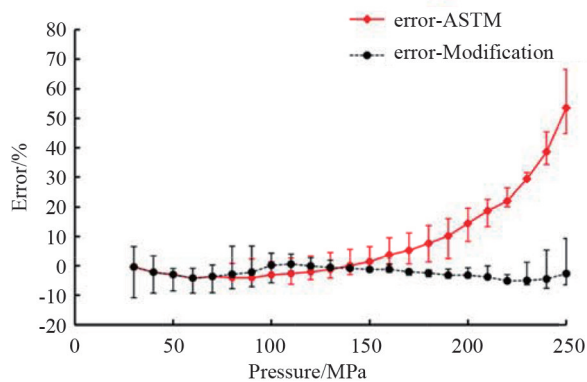


Fig. 8 Comparison of Stress Measuring Errors Before and After Plasticity Modification in the Verification Experiment

5 Conclusion

(1) Based on the distortion energy theory, calibration coefficients $\bar{a}$ and $\bar{b}$ in the plasticity deformation stage were modified and the plasticity modification formulas based on S were given.

(2) The plasticity modification formulas obtained in the study were verified by ANSYS. The simulation results show that the plasticity modification of 7075 aluminum alloy in a “thin” workpiece at high residual stress has a good effect. In the plasticity deformation stage, the variation range of the measuring errors before the plasticity modification is $-4.972\% \sim 29.398\%$, and the variation range of the measuring errors after the plasticity modification is $-4.972\% \sim -0.888\%$. The validity of the plasticity modification method is verified by simulation.

(3) The verification experiment system of uniax-

ial tensile loading test was constructed to verify the plasticity modification method. The experimental results show that the plasticity modification of 7075 aluminum alloy in a “thin” workpiece at high residual stress has a good effect, and the measuring errors after plasticity modification are significantly reduced. The variation range of the average value of the measuring errors before the plasticity modification is $-4.071\% \sim 53.440\%$, and the variation range of the average value of the measuring errors after the plasticity modification is $-5.140\% \sim 0.609\%$. The validity of the plasticity modification method is verified by experiment.

ACKNOWLEDGMENT

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