Multipath Time Delay Estimation Based on MUSIC Algorithm Under Small Sample Conditions

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Abstract: On account of the traditional multiple signal classification (MUSIC) algorithm has poor performance in time delay estimation under the condition of small sampling data and low SNR. In this paper, the traditional MUSIC algorithm is improved. The algorithm combines the idea of spatial smoothing, constructs a new covariance matrix using the covariance information of the measurement data, and constructs a weighted value using the modified noise eigenvalues to weight the traditional estimation spectrum. Simulation results show that the improved algorithm has steeper spectral peaks and better time delay resolution under the condition of inaccurate path number estimation. The time delay estimation accuracy of this algorithm is higher than that of the traditional MUSIC algorithm and the improved SSMUSIC algorithm under the conditions of small sampling data and low SNR.

Keywords: Multipath Time Delay Estimation, MUSIC Algorithm, Spatial Smoothing

1 Introduction

The time difference of arrival (TDOA) based localization technique estimates the location of a target by measuring the time difference of the signal arriving at each receiver, which has the advantages of simple architecture, high localization accuracy, and flexibility, and is of great significance for applications in modern electronic warfare[1]. The accuracy of time delay estimation directly affects the accuracy of target localization, so accurate estimation and discrimination of the time delay of the received signal is a key part of TDOA localization technology. The resolution capability of traditional time delay estimation methods based on correlation analysis is limited by bandwidth, and the performance deteriorates sharply in multipath environments[2]. Therefore, super-resolution multipath time delay estimation algorithms that break the correlation time-Riley limit are the focus of current research[3].

Multipath time delay estimation algorithms for super-resolution are currently classified into algorithms based on maximum likelihood estimation, subspace class algorithms and sparse optimization class algorithms[4-7]. Among them, the subspace class of multiple signal classification (MUSIC) algorithm was proposed by Schmid[8] in 1986 and was initially used to achieve super-resolution estimation of the Direction Of Arrival (DOA). In the literature [9] Hou and Wu first proposed that the time delay estimation problem can be transformed into a sinusoidal frequency estimation problem. The sinusoidal frequency estimation model is equivalent to the DOA estimation model; therefore, the MUSIC algorithm for DOA estimation is applicable to time delay estimation, which effectively improves the resolution of multipath time delay estimation. However, the method in the literature [9] is not effective in estimating narrowband signals and signals with slowly varying envelopes due to the inclusion of the spectral division operation[10]. Ge et al. in the literature [11] used the measurement data correlation results to construct the covariance matrix to achieve super-resolution delay estimation for signals with slowly varying envelopes, but the estimated spectrum constructed by this method is still the traditional MUSIC spectrum with However, the estimated spectrum constructed by this method is still the traditional MUSIC spectrum, which has the problems of ambiguity of the direct-path (DP) and insufficient steepness of the spectral peaks. The literature [12] uses a diagonal loading method to improve the covariance matrix, which makes the steepness of the estimated spectrum improved. However, this method is computationally complex and has poor real-time performance.

Improving the covariance matrix using spatial smoothing technique can reduce the dimensionality of the matrix calculation and make the spectral peaks steeper. The improved SSMUSIC (Signal Subspace Scaled Multiple Signal Classification) algorithm proposed in the literature [13] uses the idea of spatial smoothing to construct the covariance matrix and uses the idea of SSMUSIC algorithm to weight the estimated spectrum, and the estimation accuracy for narrowband signals and signals with uneven spectrum There is an improvement in the estimation accuracy for narrowband signals and spectrally uneven signals. However, the forward covariance matrix constructed by this method generally fails to satisfy the Hermitian matrix under the limited observation data. The literature [14] reconstructs the noise subspace to obtain a new estimation spectrum, which improves the accuracy of DOA estimation at low signal-to-noise ratio and small number of fast beats, and provides ideas for improving the weighting of the traditional estimation spectrum by making full use of the noise subspace information in this paper.

To address the problem that the traditional MUSIC time delay estimation algorithm has errors in the constructed covariance matrix due to the limited length of data, and the poor performance of time delay estimation under small sample conditions, this paper proposes an improved MUSIC algorithm. The improved MUSIC algorithm uses the spatial smoothing technique to divide the measurement data into multiple overlapping subsequences, takes the conjugate data of each subsequence, and obtains a new covariance matrix by calculation, performs eigenvalue decomposition on the new covariance matrix to obtain the noise eigenvalues, and then uses the modified noise eigenvalues to construct weighted values to weight the traditional estimation spectrum. The improved MUSIC algorithm effectively utilizes the covariance information and noise eigenvalue information of measurement data to solve the problem of poor performance of the traditional MUSIC time delay estimation algorithm under small sample conditions, and the simulation verifies its effectiveness and feasibility.

2 MUSIC Time Delay Estimation Model

In the multipath environment, the received signals of the two receivers can be expressed after sampling as

$$\begin{cases} y_1(n)=\sum_{i=1}^{D_1}\alpha_{1i}s(n-\tau_{1i})+w_1(n) \\ y_2(n)=\sum_{i=1}^{D_2}\alpha_{2i}s(n-\tau_{2i})+w_2(n) \end{cases} \quad n=0,1,...,N-1 \tag{1}$$

Where, $y_1(n)$ and $y_2(n)$ represent the multipath signals received by each of the two receivers, $s(n)$ represent the unknown source signals, $w_1(n)$ and $w_2(n)$ represent additive Gaussian white noise. $s(n)$, $w_1(n)$ and $w_2(n)$ are uncorrelated. D_1 and D_2 are the number of paths for multipath propagation. $\alpha_{1\mathrm{i}}$ and $\alpha_{2\mathrm{i}}$ represent the random amplitudes associated with the scattering characteristics and propagation attenuation and are uncorrelated. $\tau_{1\mathrm{i}}$ and $\tau_{2\mathrm{i}}$ represent the time delay of each path.

To simplify the analysis, assume that $y_1(n)$ has only direct waves, that is, D_1=1, α_{11}=1. In the passive time delay estimation, we focus on the relative time delay. Suppose τ_{11}=0, $\tau_{2\mathrm{i}}$ represents the relative time delay between the multipath component of $y_2(n)$ and the direct wave component of $y_1(n)$. Eq. (1) can be rewritten as

$$\begin{cases} y_1(n) = s(n) + w_1(n) \\ y_2(n) = \sum_{i=1}^{D_2} \alpha_{2i} s(n - \tau_{2i}) + w_2(n) \end{cases} \quad n = 0,1,\ldots,N-1 \quad (2)$$

The autocorrelation of the received signal $y_1(n)$ is

$$\begin{aligned} r_{y_1y_1}(m) &= E\left[y_1(n) y_1(n+m) \right] \\ &= E\left[s(n)s(n+m) + w_1(n) w_1(n+m) \right] \\ &= r_{ss}(m) + r_{w_1w_1}(m) \end{aligned} \quad (3)$$

Where, $r_{ss}(m)$ and $r_{w1w1}(m)$ represent the autocorrelation of $s(n)$ and $w_1(n)$, respectively. The power spectrum of $y_1(n)$ can be deduced from the Wiener-Sinchin theorem as

$$S_{y_1y_1}(w) = S_{ss}(w) + S_{w_1w_1}(w) \quad (4)$$

Where $S_{ss}(w)$ and $S_{w_1w_1}(w)$ represent the fourier transform of $r_{ss}(m)$ and $r_{w_1w_1}(m)$, respectively.

The cross correlation between the received signals $y_1(n)$ and $y_2(n)$ is

$$\begin{aligned} r_{y_1y_2}(m) &= E\left[y_1(n) y_2(n+m) \right] \\ &= E\left[s(n) \sum_{i=1}^{D_2} \alpha_{2i} s(n+m-\tau_{2i}) \right] \\ &= \sum_{i=1}^{D_2} \alpha_{2i} r_{ss}(m - \tau_{2i}) \end{aligned} \quad (5)$$

The cross power spectrum of the received signals $y_1(n)$ and $y_2(n)$ is

$$S_{y_1y_2}(w) = S_{ss}(w) \sum_{i=1}^{D_2} \alpha_{2i} e^{-jw\tau_{2i}} \quad (6)$$

Substituting Eq. (4) into Eq. (6)

$$S_{y_1y_2}(w) = \left(S_{y_1y_1}(w) - S_{w_1w_1}(w) \right) \sum_{i=1}^{D_2} \alpha_{2i} e^{-jw\tau_{2i}} \quad (7)$$

The normalized cross spectrum of received signal is obtained by normalizing $S_{y_1y_2}(w)$ with $S_{y_1y_2}(w)$

$$h(w) = \frac{S_{y_1y_2}(w)}{S_{y_1y_1}(w)} = \sum_{i=1}^{D_2} \alpha_{2i} e^{-jw\tau_{2i}} - \varepsilon(w) \quad (8)$$

Where, $\varepsilon(w) = \left(S_{w_1w_1}(w) / S_{y_1y_1}(w) \right) \sum_{i=1}^{D_2} \alpha_{2i} e^{-jw\tau_{2i}}$.

The normalized cross spectrum $h(w)$ contains the information of time delay and attenuation coefficient, so the problem of time delay estimation is transformed into the Eq. (8).

Sampling $h(w)$ in frequency domain

$$h(2\pi k / K) = \frac{S_{y_1y_2}(w)}{S_{y_1y_1}(w)} = \sum_{i=1}^{D_2} \alpha_{2i} e^{-j\frac{2\pi k}{K}\tau_{2i}} - \varepsilon(2\pi k / K) \quad (9)$$
$$k = 0,1,\ldots,K-1$$

Use $h[k]$ and $\varepsilon[k]$ to represent $h(2\pi k/K)$ and $\varepsilon(2\pi k/K)$ respectively, and Eq. (9) is rewritten as

$$h[k] = \sum_{i=1}^{D_2} \alpha_{2i} e^{-j\frac{2\pi k}{K}\tau_{2i}} - \varepsilon[k] \quad k = 0,1,\ldots,K-1 \quad (10)$$

Write Eq. (10) in vector form

$$\boldsymbol{h} = \boldsymbol{A\alpha} - \boldsymbol{\varepsilon} \quad (11)$$

Where, $\boldsymbol{h} = \left[h[0], h[1], \ldots, h[K-1] \right]^{\mathrm{T}}$,

$\boldsymbol{\varepsilon} = \left[\varepsilon[0], \varepsilon[1], \ldots, \varepsilon[K-1] \right]^{\mathrm{T}}$,

$\boldsymbol{\alpha} = \left[\alpha_{21}, \alpha_{22}, \ldots, \alpha_{2D_2} \right]^{\mathrm{T}}$,

$\boldsymbol{A} = \left[\mathrm{a}(\tau_{21}), \mathrm{a}(\tau_{22}), \ldots, \mathrm{a}(\tau_{2D_2}) \right]$,

$\mathrm{a}(\tau_{2i}) = \left[1, e^{-j2\pi\tau_{2i}/K}, \ldots, e^{-j2\pi\tau_{2i}(K-1)/K} \right]^{\mathrm{T}}$.

Considering $h[k]$ as the array received data in DOA estimation, Eq. (10) is internally consistent with the DOA estimation model under uniform line array (ULA) structure, so the MUSIC algorithm in DOA estimation can be applied to multipath time delay estimation.

3 Super-resolution Multipath Time Delay Estimation

3.1 Traditional MUSIC Time Delay Estimation Method

When using the traditional MUSIC algorithm to estimate the multipath delay, first calculate the covariance matrix $\boldsymbol{R}$ of the normalized cross-spectrum $\boldsymbol{h}$, and then perform eigenvalue decomposition on the matrix $\boldsymbol{R}$. The signal subspace is composed of the eigenvectors corresponding to the top D_2 (In this paper, the multipath number D_2 is estimated by the minimum description length criterion)[15] largest eigenvalues, and the noise subspace is composed of the remaining eigenvectors. Finally, the estimated spectrum is constructed using the orthogonality of the signal subspace and the noise subspace, and the estimated spectrum is searched for spectral peaks, and the position corresponding to the peak is the estimated time delay value.

The covariance matrix $\boldsymbol{R}$ of the normalized cross-spectrum $\boldsymbol{h}$ is given by

$$\begin{aligned}\boldsymbol{R} &= E\left[\boldsymbol{h}\boldsymbol{h}^{\mathrm{H}}\right] = E\left[\left(\boldsymbol{A}\boldsymbol{\alpha}+\boldsymbol{\varepsilon}\right)\left(\boldsymbol{\alpha}^{\mathrm{H}}\boldsymbol{A}^{\mathrm{H}}+\boldsymbol{\varepsilon}^{\mathrm{H}}\right)\right] \\ &= \boldsymbol{A}E\left[\boldsymbol{\alpha}\boldsymbol{\alpha}^{\mathrm{H}}\right]\boldsymbol{A}^{\mathrm{H}} + E\left[\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^{\mathrm{H}}\right] \\ &= \boldsymbol{A}\boldsymbol{P}\boldsymbol{A}^{\mathrm{H}} + \sigma^2\boldsymbol{I}\end{aligned} \tag{12}$$

Where $\boldsymbol{P}=E[\boldsymbol{\alpha}\boldsymbol{\alpha}^{\mathrm{H}}]$. Assuming that the amplitude of the attenuation coefficient of each path is constant, and the phase follows a uniform distribution in [0:1], then P is a non-singular matrix. Since the delay value of each path is different, the matrix $\boldsymbol{A}$ is a full-rank matrix, and $\boldsymbol{R}$ is a non-singular symmetric matrix. The eigenvalues of $\boldsymbol{R}$ are decomposed, the eigenvectors corresponding to the top D_2 largest eigenvalues constitute the signal subspace, and the remaining eigenvectors constitute the noise subspace. The noise subspace is represented by $\hat{\boldsymbol{U}}_N$. Since the signal subspace and the noise subspace are orthogonal, the vector $\mathrm{a}(\tau_{2i})$ falls in the signal subspace, so $\mathrm{a}^{\mathrm{H}}(\tau_{2i})\hat{\boldsymbol{U}}_N = 0$. Therefore, the expression of the MUSIC time delay estimation spectrum is

$$P_{MUSIC}(\tau) = \frac{1}{\mathrm{a}^{\mathrm{H}}(\tau)\hat{\boldsymbol{U}}_N\hat{\boldsymbol{U}}_N^{\mathrm{H}}\mathrm{a}(\tau)} \tag{13}$$

Perform spectral peak search on Eq. (13), and obtain the time $\boldsymbol{\tau}$ corresponding to the top D_2 maximum peak points, which is the estimated multipath time delay value.

3.2 Improved MUSIC Time Delay Estimation Method

In the MUSIC algorithm, the covariance matrix needs to be calculated from the statistics of multiple measurement data. In practical applications, the measurement data value samples are limited, and the covariance matrix generated at this time has errors, which leads to the phenomenon of insufficient rank in the eigenvalue decomposition, thus making it difficult to distinguish signal eigenvalues from noise eigenvalues and making the performance of MUSIC algorithm unstable. In this regard, the spatial smoothing technique can be used to improve the covariance matrix. The basic idea of spatial smoothing technique is to divide the main array into several overlapping sub-arrays, and then average the covariance matrix of the sub-arrays to solve the correlation of the signal causing rank deficit by spatial smoothing. In this paper, the technique is migrated to time delay estimation.

In order to ensure the real-time performance of delay estimation, it is supposed that a normalized cross-spectral sequence of length K is obtained at one time, and spatial smoothing is to serialize the data into overlapping subsequences of length M, and let

$$\begin{gathered}\boldsymbol{h}_q = \left[h[q], h[q+1], \ldots, h[q+M-1]\right]^{\mathrm{T}} \\ q = 0,1,\ldots,K-M\end{gathered} \tag{14}$$

Where, the number of subsequences is $K–M+1$. The mean of the covariance matrix of all subsequences is the forward covariance matrix, and its expression is as follows

$$\hat{\boldsymbol{R}}_1 = \frac{1}{K-M+1}\sum_{q=0}^{K-M}\boldsymbol{h}_q\boldsymbol{h}_q^{\mathrm{H}} \tag{15}$$

Under ideal conditions, the amplitude of the attenuation coefficient of each path is constant, and the phase obeys a uniform distribution in $[0,2\pi]$. At this time, $\hat{\boldsymbol{R}}_1$ satisfies the Hermitian matrix. However, when the sampled data is limited, the obtained $\hat{\boldsymbol{R}}_1$ generally does not satisfy this property. Therefore, a new covariance matrix is introduced in this paper.

Take the conjugate form $\boldsymbol{h}_q^*$ of each subsequence data $\boldsymbol{h}_q$, and define a new data vector $\boldsymbol{x}_q$

$$\boldsymbol{x}_q = \boldsymbol{J}\boldsymbol{h}_q^* \tag{16}$$

Where, J is the $M\times M$ order exchange matrix with 1 element on the antidiagonal and the remaining elements are 0. Find the mean $\hat{\boldsymbol{R}}_2$ of the autocorrelation matrix of $\boldsymbol{x}_q$, and the mean $\hat{\boldsymbol{R}}_3$ of the cross-correlation matrix of $\boldsymbol{h}_q$ and $\boldsymbol{x}_q$

$$\begin{aligned}\hat{\boldsymbol{R}}_2 &= \frac{1}{K-M+1}\sum_{q=0}^{K-M}\boldsymbol{x}_q\boldsymbol{x}_q^{\mathrm{H}} \\ &= \frac{1}{K-M+1}\sum_{q=0}^{K-M}\boldsymbol{J}\boldsymbol{h}_q^*\left(\boldsymbol{J}\boldsymbol{h}_q^*\right)^{\mathrm{H}} \\ &= \frac{1}{K-M+1}\sum_{q=0}^{K-M}\boldsymbol{J}\left(\boldsymbol{h}_q\boldsymbol{h}_q^{\mathrm{H}}\right)^*\boldsymbol{J} \\ &= \boldsymbol{J}\hat{\boldsymbol{R}}_1^*\boldsymbol{J}\end{aligned} \tag{17}$$

$$\hat{\boldsymbol{R}}_3 = \frac{1}{K-M+1}\sum_{q=0}^{K-M}\boldsymbol{h}_q\boldsymbol{x}_q^{\mathrm{H}} \tag{18}$$

The new covariance matrix R is defined as:

$$\hat{\boldsymbol{R}} = \frac{\hat{\boldsymbol{R}}_1+\hat{\boldsymbol{R}}_2+\hat{\boldsymbol{R}}_3+\boldsymbol{J}\hat{\boldsymbol{R}}_3^*\boldsymbol{J}}{4} \tag{19}$$

The new covariance matrix $\hat{\boldsymbol{R}}$ effectively utilizes the conjugate information of the measurement data, satisfies $\boldsymbol{J}\hat{\boldsymbol{R}}\boldsymbol{J} = \hat{\boldsymbol{R}}^*$. Its elements are conjugate symmetric on the main diagonal[16], and it more satisfies the properties of Hermitian matrix. In the formula, the value of M directly affects the spectral peak search effect. This paper studies the effect of M on the peak to average ratio and main lobe width of spectral peak search through simulation experiments, and determines the appropriate value. The experimental results and analysis are given in 4.1.

Theoretically, when the background noise is Gaussian white noise, the noise eigenvalues are all equal to σ^2. In practice, there are errors in the generated covariance matrix, so the noise eigenvalues obtained from the decomposition are not exactly the same. In this regard, this paper uses the corrected noise eigenvalues to design a weight matrix to weight the estimated spectra and equalize the differences between them.

Perform eigenvalue decomposition on the covariance matrix $\hat{\boldsymbol{R}}$ and arrange the resulting eigenvalues in descending order

$$\hat{\lambda}_1 \geqslant \hat{\lambda}_2 \geqslant \ldots \geqslant \hat{\lambda}_{D_2} \geqslant \hat{\lambda}_{D_2+1} > \ldots > \hat{\lambda}_M \tag{20}$$

Correct the $M-D_2$ noise eigenvalues

$$\tilde{\lambda}_i = \lambda_i + \beta \quad i = D_2+1, D_2+2, \ldots M \tag{21}$$

Where, $\tilde{\lambda}_i$ is the corrected noise eigenvalue, and β is the corrected value, which is used to control the divergence degree between the noise eigenvalues. Using $1/\tilde{\lambda}_i$ as the weight to assign to the estimated spectrum, the expression of the new MUSIC spectrum is

$$P_{MUSIC}(\tau) = \frac{1}{\sum_{i=D_2+1}^{M}\frac{1}{\tilde{\lambda}_i}\left|\mathrm{a}^{\mathrm{H}}(\tau)\boldsymbol{v}_i\right|^2} \tag{22}$$

Where, v_i is the eigenvector corresponding to the eigenvalue λ_i of the noise. Compared with Eq. (13) and Eq. (22), Eq. (13) can be regarded as using an equal weight matrix. When the SNR is low, the noise eigenvalues diverges and the gap between the noise eigenvalues and the signal eigenvalues becomes smaller. Eq. (22) uses the corrected noise eigenvalues to construct a weighted value, changes the weight of the projection component of a(τ) in the noise subspace, and equalizes the difference between the noise eigenvalues. It can reduce noise interference when parameter estimation is inaccurate.

For the selection of β, the literature [17] used the information theory criterion, and through a large number of empirical data analysis, it was found that the ratio of the maximum value to the minimum value of noise eigenvalue is less than 2 when the number of information sources can be correctly estimated. Substitute the modified noise eigenvalue

$$\frac{\lambda_{D_2+1}+\beta}{\lambda_M+\beta} \leqslant 2 \tag{23}$$

This paper selects the smallest β that satisfies Eq. (23).

4 Simulations and Analysis

In this paper, the linear frequency modulation signal is used as the transmission signal for simulation experiments [18], and its expression is as follows

$$\begin{gathered}s(n) = \sin\left(\left(\xi\cdot n+\zeta\right)\cdot n+\varphi_0\right) \\ n = 0,\ 1,\ \ldots,\ N-1\end{gathered} \tag{24}$$

Where, $\xi=\pi\cdot(f_2-f_1)/N$, f_1 and f_2 represent the lowest and highest frequencies of the signal, respectively. $\zeta=2\pi\cdot f_1.\varphi_0$ represents the random initial phase of the signal. The simulation settings f_1=0.3, f_2=0.5. The defined bandwidth is $B_s \triangleq f_2 - f_1$, and the number of multipaths D_2 is set to 2. The delay of signal is τ_{21}=5, τ_{22}=8, and the delay difference between the two signals is $\Delta\tau = |\tau_{21} - \tau_{22}| = 3 < \frac{1}{B_s}$, which belongs to the super-resolution situation. The number of data points N=48, the normalized cross-spectral sequence length K=95, and the search step length is 0.01.

4.1 Selection of Subsequence Length M

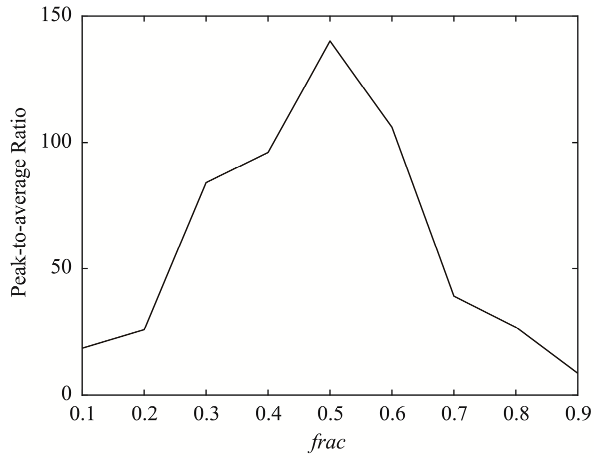
The simulation settings α_{21}=1, α_{22}=0.8, SNR is –8*dB*, subsequence length M=[*frac*×K], where $\lfloor x \rfloor$ represents downward rounding, and the ratio *frac*=[0.2:0.1:0.9]. The peak-to-average ratio and main lobe width are used to measure the peak searching effect under different M, and 100 Monte Carlo simulation experiments are carried out. Main lobe width is defined as the distance between the first two points around the peak that reach 20% of the peak height. The variation of peak-to-average ratio and main lobe width with the ratio of M to K are shown in Fig.1. From Fig.1a, it can be seen that the peak-to-average ratio first increases and then decreases with the increase of *frac*. The maximum value is obtained at *frac*=0.5. From Fig.1b, it can be seen that the main lobe width decreases first and then increases with the increase of *frac*. The minimum value is obtained at *frac*=0.5. Considering comprehensively, *frac*=0.5 is selected as the length of subsequence in this paper. At this time, the spectral peak search effect is better.

4.2 Normalized Spectrum Before and After Improvement of Covariance Matrix

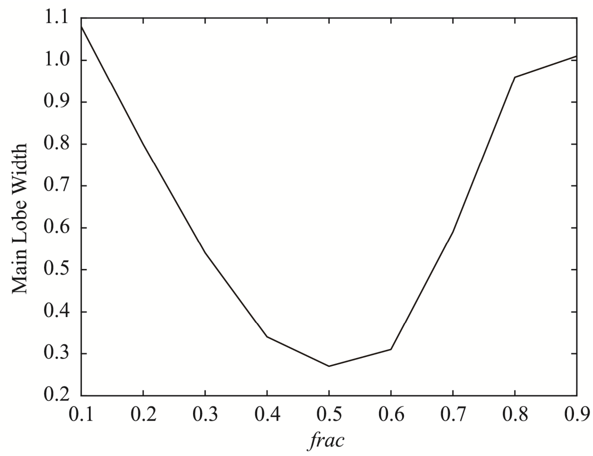
The simulation settings α_{21}=1, α_{22}=0.8, and the SNR is –8*dB*. Fig.2 shows the MUSIC normalized spectrum before and after using the method in this paper to improve the covariance matrix. Comparing Fig.2a and Fig.2b, it can be seen that the spectral peak of the improved MUSIC normalized spectrum is steeper, the spectral line is smoother at the non-delay time point, and the time delay estimation accuracy is higher.

4.3 Normalized Spectrum Before and After Weighting

The simulation settings α_{21}=0.25, α_{22}=1, and SNR is –11*dB*. At this time, the number of multipaths estimated by the minimum description length criterion is 1.



(a)



(b)

Fig.1 Variation of Peak-to-average Ratio and Main Lobe Width with *frac* (a) Variation of Peak-to-average Ratio Width with *frac* (b) Variation of Main Lobe Width with *frac*

After improving the covariance matrix, use the weight matrix designed in this paper to weight the MUSIC spectrum. The effect before and after weighting is shown in Fig.3. Comparing Fig.3a and Fig.2b, under the condition of inaccurate estimation of the number of paths, there is only one peak in the MUSIC spectrum before weighting, and the weighted MUSIC spectrum has two peaks. The first path can be distinguished after weighting. It shows that the weighted improvement can reduce noise interference to some extent, and the improved MUSIC algorithm has better time delay resolution.

4.4 Performance Comparison of Three Algorithms Under Different SNR

Under the condition of N=48 and SNR=–10:2:4dB, the performance of time delay estimation of the improved MUSIC algorithm is compared with that of the traditional MUSIC algorithm and SSMUSIC algorithm. The number of multipath D_2=2 and D_2=4 are considered respectively. When D_2=2, the simulation settings $\alpha_{21}=1$, $\tau_{21}=5$; $\alpha_{22}=0.8$, $\tau_{22}=8$. When D_2=4, two signals, $\alpha_{23}=0.6$, $\tau_{23}=11$ and $\alpha_{24}=0.5$, $\tau_{24}=14$, are added. Conduct

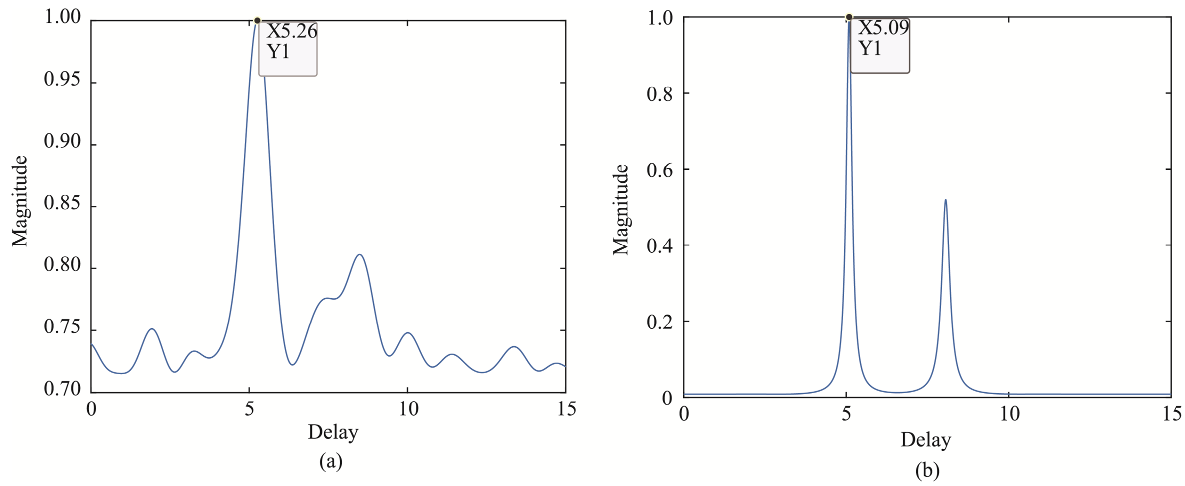


Fig.2 Multipath Time Delay Estimation Based on Music Algorithm Under Small Sample Conditions Normalized Spectrum Before and After Improvement of Covariance Matrix (a) Normalized Spectrum Before Improvement of Covariance Matrix (b) Normalized Spectrum After Improvement of Covariance Matrix

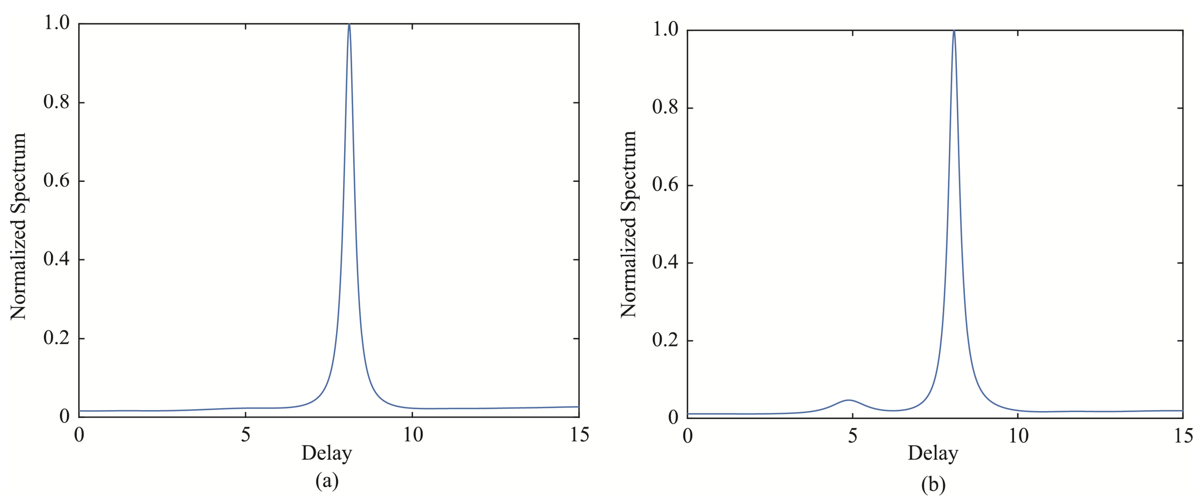


Fig.3 Normalized Spectrum Before and After Weighting (a) Normalized Spectrum Before Weighting (b) Normalized Spectrum After Weighting

100 Monte Carlo simulation experiments. The variation of root mean square error (RMSE) of the three algorithms with SNR is shown in Fig.4. From Fig.4, it can be seen that the time delay estimation performance of each algorithm improves with the increase of SNR. Compared with the improved MUSIC algorithm and SSMUSIC algorithm, which use spatial smoothing preprocessing technology, the performance of traditional MUSIC algorithm is poor. The RMSE of the improved MUSIC algorithm is smaller than that of the SSMUSIC algorithm under different signal-to-noise ratios, when the number of multipath is the same. In the case of D_2=2 and $SNR \leqslant -4dB$, the RMSE of the improved MUSIC algorithm is more than 0.01 smaller than that of the SSMUSIC algorithm. Therefore, the time delay estimation performance of the algorithm in this paper is superior to the traditional MUSIC algorithm and SSMUSIC algorithm.

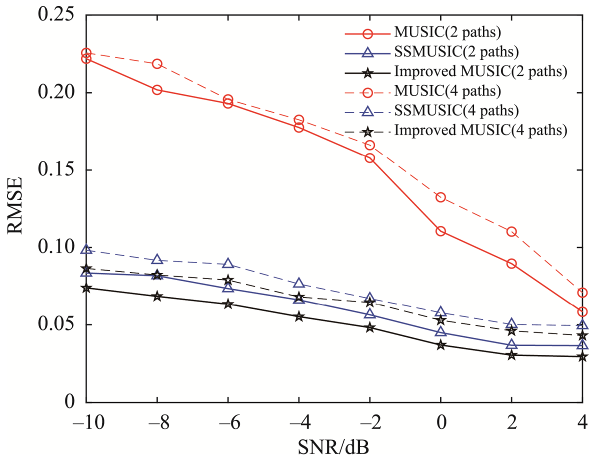


Fig.4 Simulation Results Under Different SNR

4.5 Performance Comparison of Three Algorithms Under Different Calculation Points

Under the condition of SNR=–6dB and calculation points N=20 : 10 : 100, the time delay estimation performance of the improved MUSIC algorithm is compared with that of the traditional MUSIC algorithm and SSMUSIC algorithm. Considering the number of multipath D_2=2 and D_2=4 respectively, the signal delay and attenuation coefficient are the same as 4.4. Conduct 100 Monte Carlo simulation experiments. The RMSE of the three algorithms varies with the number of calculation points as shown in Fig.5. From Fig.5, it can be seen that the RMSE of the three algorithms decreases with the increase of calculation points. The RMSE of this algorithm is smaller than the traditional MUSIC algorithm and the improved SSMUSIC algorithm under different data points, when the number of multipath is the same. In the case of D_2=2 and $N \geqslant 30$, the improved MUSIC algorithm can control the RMSE within 0.1. Therefore, the algorithm in this paper has more advantages under the condition of small samples.

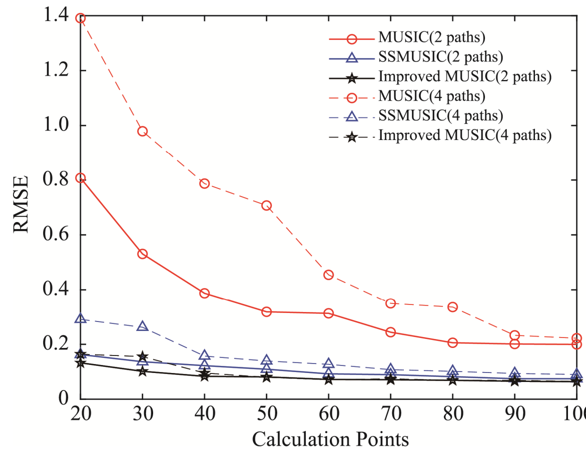


Fig.5 Simulation Results Under Different Calculation Points

5 Conclusion

This paper firstly studies the MUSIC time delay estimation model, and applies the MUSIC algorithm in DOA estimation to multipath time delay estimation. To address the problem that the traditional MUSIC delay estimation algorithm has poor performance in delay estimation under small sample conditions, the covariance matrix and estimation spectrum of the traditional MUSIC algorithm are improved, and a super-resolution multipath time delay estimation method based on the improved MUSIC algorithm is proposed, and simulation experiments are conducted. The improved MUSIC algorithm, combined with the idea of spatial smoothing, uses the covariance information of measurement data to construct a new covariance matrix, and the new covariance matrix better satisfies the properties of Hermitian matrix under non-ideal conditions; the modified noise eigenvalues are used to construct weighted values to weight the traditional estimation spectrum, which equalizes the differences between noise eigenvalues and reduces the noise interference. The improved algorithm has steeper spectral peaks and better delay resolution under the conditions of low SNR and inaccurate path number estimation, and the performance of time delay estimation under different SNR and different calculation points is better than the traditional MUSIC algorithm and the improved SSMUSIC algorithm.

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