# Study of high-precision earth sensor with triple-FOV

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**Abstract**: Earth sensors are widely used in spacecraft for attitude determination. They need to have a very large field of view  $(FOV)(>120^{\circ})$  and relatively low accuracy while being used in the aircrafts around orbit. A triple-FOV infrared earth sensor is proposed in this paper. It uses three pieces of standard infrared detectors with a wavelength range of  $14 \sim 16\mu$ m, to sense the horizontal circle by detecting the infrared light emitted from the earth. From which, the geocentric vector can be obtained. A mathematic model is established and a validation model is set up to provide input parameters for the mathematic model. The simulation results of the two models show that the output of the mathematic model coincides with the known parameters. Based on the above analysis, a prototype has been built and tested. The test results show that the angle measurement error is about 0.002° and hence such a triple-FOV earth sensor is capable to provide high-precision position information for autonomous navigation. **Key words**; Earth Sensor, Validation Model, High Precision, Autonomous Navigation

#### 1 Introduction

Earth sensor is an important subsystem of the attitude determination and control systems used in spacecraft.<sup>[1-3]</sup> It observes the discontinuity between the earth and the space to obtain the horizontal circle. Then, the geocentric vector can be determined <sup>[4-7]</sup>. It is well known that infrared earth sensors can be divided into two categories, static ones and dynamic scanning ones, based on their working principle. Although the technology of dynamic scanning earth sensors <sup>[8-10]</sup> is relatively mature, its life expectancy and development are subject to various restrictions. While static earth sensors have the advantages in quality, power, accuracy and life expectancy. Thus, it has gradually become the focus of research. <sup>[11-14]</sup>

A triple-FOV earth sensor based on standard infrared detectors is proposed to meet the requirement of being used in aircrafts around earth orbit. It can work out the geocentric vector and calculate the altitude of an aircraft by sensing the horizontal circle through the atmosphere radiation in the wavelength range of  $14 \sim 16 \mu m$ , which can make sure that such an earth sensor is capable to operate normally day and night regardless of the weather condition and the light impact <sup>[15-17]</sup>.

## 2 Mathematic Model

A mathematic model of triple-FOV earth sensor is established based on the use of linear CCDs. As shown in Figure 1, three optical axes are set as  $120^{\circ}$ from each other, and  $q_0$  is a structural parameter between the optical axis and  $z_{es}$ .

As shown in Figure 2, the initial coordinate frame is set as  $x_1y_1z_1$ . Then, the coordinate frame rotates in turn of z-axis, x-axis and y-axis with an angle of  $\alpha$ ,  $\beta$  and  $\gamma$ , respectively. Finally,  $x_2y_2z_2$  and the coordinate transformation matrix can be calculated.

Considering that the vector to the Earth centre is  $\vec{E}_{11} = [0, 0, -1]^{T}$  in  $x_1 y_1 z_1$  frame, the geocentric vector in the new coordinate frame can be expressed as follows by using the transformation matrix, while  $\alpha$  can be set as 0.

$$\vec{E}_{21} = [\cos\beta\sin\gamma - \sin\beta - \cos\beta\cos\gamma]^{\mathrm{T}}$$
(1)



Fig. 1 Earth sensor coordinate frame



Fig. 2 Mathematic model

The center vector  $\vec{E}_{22}$  in  $x_2y_2z_2$  frame can be expressed by three vectors,  $\vec{X}_{21} \vec{X}_{22} \vec{X}_{23}$ , from the earth sensor to the edge. Upon the analysis above, the following equation can be obtained.

$$\vec{E}_{21} = \vec{E}_{22} \tag{2}$$

So  $\beta$  and  $\gamma$  can be expressed as follows.  $\sin\beta = -\vec{E}_{22}(2)$ 

$$\tan\gamma = -\frac{\vec{E}_{22}(1)}{\vec{E}_{22}(3)}$$
(3)

Thus, h can be expressed as follows.

$$h = \frac{R + r}{\sqrt{1 - (\cos\beta)^2 \cdot (\cos(\theta_0 - \varphi_1 - \gamma))^2}} - R \quad (4)$$

where  $\varphi_1 = \arctan(\frac{u_1}{f})$ ,  $u_1$  the signal point in the CCD1 coordinate frame, r the thickness of the carbon dioxide absorption belt in the  $14 \sim 16 \ \mu m$  range, R is the radius of the earth.

# **3** Validation Model

As shown in Figure 3, the initial state of the earth sensor is set as a fixed coordinate frame  $o_g - x_g y_g z_g$ . After rotating at the turn 3-1-2, it becomes a new coordinate frame  $o_{es} - x_{es} y_{es} z_{es}$ , where  $o_g F_1 B_1 F_2$  is the conical surface of observing the earth infrared horizon by earth sensor.

The relationship between the altitude h and half cone angle q can be expressed as follows:

$$\sin\theta = \frac{R+r}{R+h} \tag{6}$$

The conical surface equation can be expressed as follows:

$$z = -k\sqrt{x^2 + y^2}$$
(7)  
where  $k = \cot\theta$ .

The extension lines of three linear detectors intersect at one point in  $z_{es}$  axis.  $o_{es}o_{f1}$  is a parameter and set as  $o_{es}o_{f1} = o_{es}o_{f2} = o_{es}o_{f3} = a$ .



Fig. 3 Validation model

As shown in Figure 3, the linear parametric equations  $p_g o_{f1,g}$ ,  $p_g o_{f2,g}$  and  $p_g o_{f3,g}$  can be obtained as follows:

$$\begin{cases} x = t_m (x_{ofm,g} - x_{pg}) + x_{pg} \\ y = t_m (y_{ofm,g} - y_{pg}) + y_{pg} \\ z = t_m (z_{ofm,g} - z_{pg}) + z_{pg} \end{cases}$$
(8)  
where tm>0 and m=1, 2, 3.

After solving the equations between the linear parametric equations and the conical surface equation, the signal point  $A'_{m,g}$  (m=1,2,3) can be obtained.

However, the signal point  $A_{m,g}$  can also be expressed after the coordinate transformation from  $A_{m,fm}$  in the CCD coordinate frame  $f_m$  (m=1,2,3).

Since  $A'_{m,g}$  is equal to  $A_{m,g}$ , the signal point coordinates  $u_1 u_2 u_3$  in the CCD coordinate frame can be obtained as follows.

$$u_{1} = -\frac{\left[t_{1}\left(x_{of1,g} - x_{pg}\right) + x_{pg} + a\cos\beta\cos\left(\theta_{0} - \gamma\right)\right]}{\sin\theta_{0}\cos\gamma - \cos\theta_{0}\sin\gamma}$$

$$u_{2} = -\frac{\left[t_{2}\left(x_{of2,g} - x_{pg}\right) + x_{pg} - \cos\left(\frac{2\pi}{3}\right)a\sin\theta_{0}\cos\gamma + a\cos\theta_{0}\sin\gamma\right]}{\cos\left(\frac{2\pi}{3}\right)\cos\theta_{0}\cos\gamma + \sin\theta_{0}\sin\gamma}$$

$$u_{3} = -\frac{\left[t_{3}\left(x_{of3,g} - x_{pg}\right) + x_{pg} - \cos\left(\frac{4\pi}{3}\right)a\sin\theta_{0}\cos\gamma + a\cos\theta_{0}\sin\gamma\right]}{\cos\left(\frac{4\pi}{3}\right)\cos\theta_{0}\cos\gamma + \sin\theta_{0}\sin\gamma}$$
(9)

## 4 Simulation Analysis

#### 4.1 Simulation between two models

Among the parameters in use, h changes from 50km to 100km, r is set to be 20km, R is 6370km, a is 0.05m, and  $\beta$  and  $\gamma$  change from -10° to +10° respectively. With the above parameters as the input to the validation model,  $u_1$ ,  $u_2$ ,  $u_3$  can be obtained correspondingly. Figure 4 to Figure 6 show that  $u_1$ ,  $u_2$  and  $u_3$  change with  $\beta$ ,  $\gamma$  and h, respectively.



Fig. 4  $u_1$  changes with  $\beta$ ,  $\gamma$  and h



Fig. 6  $u_3$  changes with  $\beta$ ,  $\gamma$  and h

Take  $u_1$ ,  $u_2$  and  $u_3$  as input parameters of the mathematic model. The altitude h, roll angle  $\beta$  and pitch angle  $\gamma$  can be obtained respectively. Figure 7 to Figure 9 show that h,  $\beta$  and  $\gamma$  change with  $u_1$ ,  $u_2$  and  $u_3$ , respectively.



Fig. 7 h changes with  $u_1$ ,  $u_2$  and  $u_3$ 



Fig. 8  $\beta$  changes with  $u_1$ ,  $u_2$  and  $u_3$ 



Fig. 9  $\gamma$  changes with  $u_1$ ,  $u_2$  and  $u_3$ 

The simulation results show that the output of mathematic model coincides with the known parameters well, that is, the validation model can provide suitable input parameters for the mathematic model.

#### 4.2 Detection accuracy simulation

When the extraction points on the detectors have errors,  $u_1$ ,  $u_2$  and  $u_3$  will have an effect on the results of height measurement and attitude determination. Assuming that the signal extraction error is 0.1 pixel, there are eight situations as shown in Table 1, where  $u_{10}$ ,  $u_{20}$  and  $u_{30}$  are known points.

The results of  $\beta$ ,  $\gamma$  and h are shown in Table 2, where the measurement error of h is about 13.358m and the attitude measurement error is about 0.001°.

Table 1 Eight situations of  $u_1$ ,  $u_2$  and  $u_3$  with error

	<b>u</b> <sub>1</sub>	u <sub>2</sub>	u <sub>3</sub>
1	$u_{10}$ + 0.1 pixel	$u_{20}$ + 0.1 pixel	$u_{30}$ + 0.1 pixel
2	$u_{10}$ + 0.1 pixel	$u_{20}$ + 0.1 pixel	u <sub>30</sub> - 0.1 pixel
3	$u_{10}$ + 0.1 pixel	$u_{20}$ - 0.1 pixel	u <sub>30</sub> + 0.1 pixel
4	$u_{10}$ + 0.1 pixel	$u_{20}$ - 0.1 pixel	u <sub>30</sub> - 0.1 pixel
5	u <sub>10</sub> - 0.1 pixel	$u_{20}$ + 0.1 pixel	u <sub>30</sub> + 0.1 pixel
6	$u_{10}$ - 0.1 pixel	$u_{20}$ + 0.1 pixel	u <sub>30</sub> - 0.1 pixel
7	$u_{10}$ - 0.1 pixel	$u_{20}$ - 0.1 pixel	u <sub>30</sub> + 0.1 pixel
8	u <sub>10</sub> - 0.1 pixel	u <sub>20</sub> - 0.1 pixel	u <sub>30</sub> - 0.1 pixel

Table 2  $\beta$ ,  $\gamma$  and h change with different situations of  $u_1$ ,  $u_2$  and  $u_3$ 

	β∕°	γ∕°	h/m
1	0.0000	0.0000	13.3583
2	0.0011	-0.0006	4.7473
3	-0.0011	-0.0006	4.7473
4	0.0000	-0.0012	-3.8629
5	0.0000	0.0012	4.7473
6	0.0011	0.0006	-3.8629
7	-0.0011	0.0006	-3.8629
8	0.0000	0.0000	-12.4724

## 5 Test

Based on the analysis above, a test is conducted to confirm the validation model and the accuracy of the earth sensor. As shown in Figure 10, the earth sensor is fixed on the turntable. The brightness of the collimator was adjusted, and the turntable was rotated to make sure that the earth sensor can detect the incident ray from the collimator.



Fig. 10 Experiment setup

As the turntable rotates from  $-0.498^{\circ}$  to  $0.500^{\circ}$ with  $0.002^{\circ}$  step for each rotation, the relationship between the gray values and different pixels of the linear CCD is shown in Figure 11. After normalizing the gray values, linear fitting is applied to each curve and the results are shown in Figure 12.



Fig. 11 The relationship between gray values and different pixels



Taking the gray values of  $1845 \sim 1855$  pixels of the first curve as a reference, pixel values of the other linear fitting curves at the same gray value are calculated and the pixel difference between the adjacent curves are obtained. The averaged value and standard deviation of the pixel difference at a gray value are shown in Table 3, where the average is about 0.2 pixel and the standard deviation is about 0.05 pixel respectively.

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Dival	Average of pixel	Standard deviation of
I IXCI	difference	pixel difference
1845	0.2053	0.0464
1846	0.2055	0.0443
1847	0.2048	0.0436
1848	0.2047	0.0440
1849	0.2040	0.0456
1850	0.2043	0.0483
1851	0.2039	0.0514
1852	0.2030	0.0550
1853	0.2002	0.0577
1854	0.1976	0.0584
1855	0.1956	0.0608
Average	0.2026	0.0505

 
 Table 3
 The averaged value and standard deviation of pixel difference between the adjacent curves

 $0.002^{\circ}$ , the signal point will change about 0.2 pixel.

As a result, when the earth sensor rotates

## 6 Conclusion

Aiming for the use in aircrafts around the earth orbit, a triple-FOV earth sensor is proposed and simulated. A validation model is established to provide input parameters for the earth sensor and a way to test the earth sensor. A prototype is built and tested in laboratory. The simulation and testing results show that the attitude measurement error is about  $0.002^{\circ}$ . This shows that the earth sensor proposed in this paper can provide high-precision information for aircrafts to achieve autonomous navigation.

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